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Mathematics: analysis and approaches
Standard level
Paper 1

Thursday 6 May 2021 (afternoon)

Candidate session number

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1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

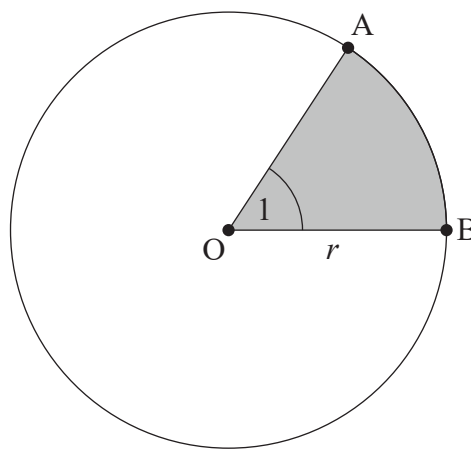
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius r .

diagram not to scale



Points A and B lie on the circumference of the circle, and $\widehat{AOB} = 1$ radian .

The perimeter of the shaded region is 12.

(a) Find the value of r . [3]

(b) Hence, find the exact area of the **non-shaded** region. [3]

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3. [Maximum mark: 6]

(a) Show that the equation $2 \cos^2 x + 5 \sin x = 4$ may be written in the form $2 \sin^2 x - 5 \sin x + 2 = 0$.

[1]

(b) Hence, solve the equation $2 \cos^2 x + 5 \sin x = 4$, $0 \leq x \leq 2\pi$.

[5]

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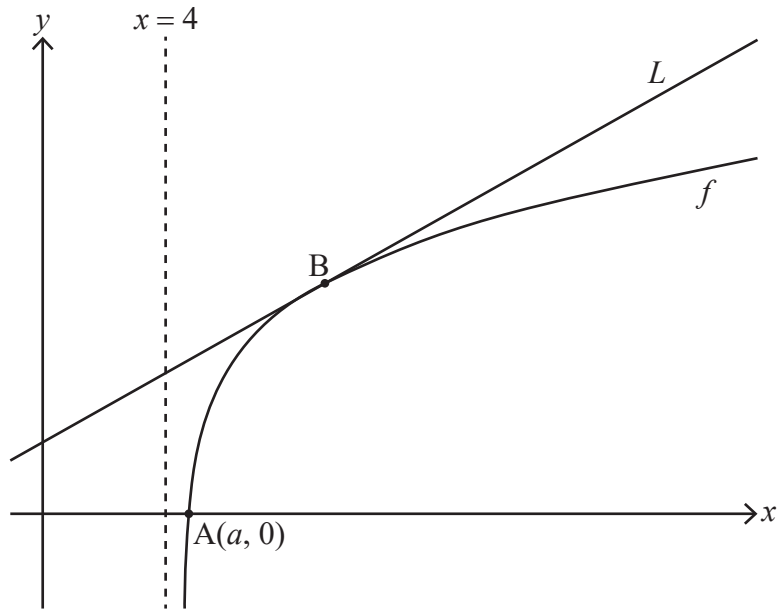
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5. [Maximum mark: 9]

Consider the function f defined by $f(x) = \ln(x^2 - 16)$ for $x > 4$.

The following diagram shows part of the graph of f which crosses the x -axis at point A, with coordinates $(a, 0)$. The line L is the tangent to the graph of f at the point B.



(a) Find the exact value of a . [3]

(b) Given that the gradient of L is $\frac{1}{3}$, find the x -coordinate of B. [6]

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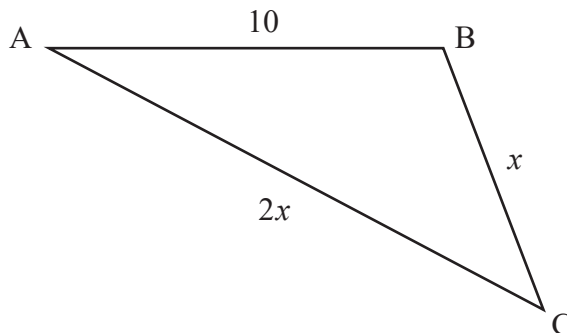
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6. [Maximum mark: 7]

The following diagram shows triangle ABC, with $AB = 10$, $BC = x$ and $AC = 2x$.

diagram not to scale



Given that $\cos \hat{C} = \frac{3}{4}$, find the area of the triangle.

Give your answer in the form $\frac{p\sqrt{q}}{2}$ where $p, q \in \mathbb{Z}^+$.

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Turn over

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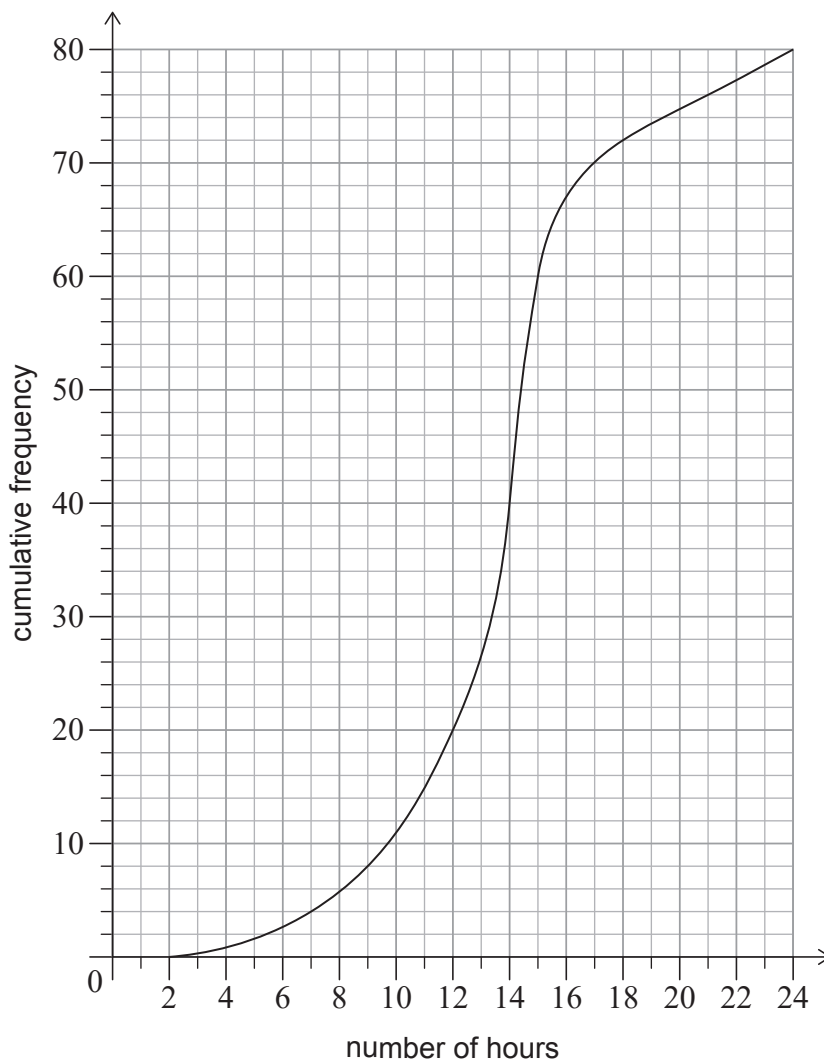
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

A large school has students from Year 6 to Year 12.

A group of 80 students in Year 12 were randomly selected and surveyed to find out how many hours per week they each spend doing homework. Their results are represented by the following cumulative frequency graph.



(This question continues on the following page)



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(Question 7 continued)

- (a) Find the median number of hours per week these Year 12 students spend doing homework. [2]
- (b) Given that 10% of these Year 12 students spend more than k hours per week doing homework, find the value of k . [3]

This same information is represented by the following table.

| | | | | |
|--|----------------|-----------------|------------------|------------------|
| Hours (h) spent doing homework | $2 < h \leq 7$ | $7 < h \leq 15$ | $15 < h \leq 21$ | $21 < h \leq 24$ |
| Frequency | 4 | p | 16 | q |

- (c) Find the value of p and the value of q . [4]

There are 320 students in Year 12 at this school.

- (d) Estimate the number of Year 12 students that spend more than 15 hours each week doing homework. [3]
- (e) (i) Explain why this sampling method might not provide an accurate representation of the amount of time **all** of the students in the school spend doing homework.
(ii) Suggest a more appropriate sampling method. [2]

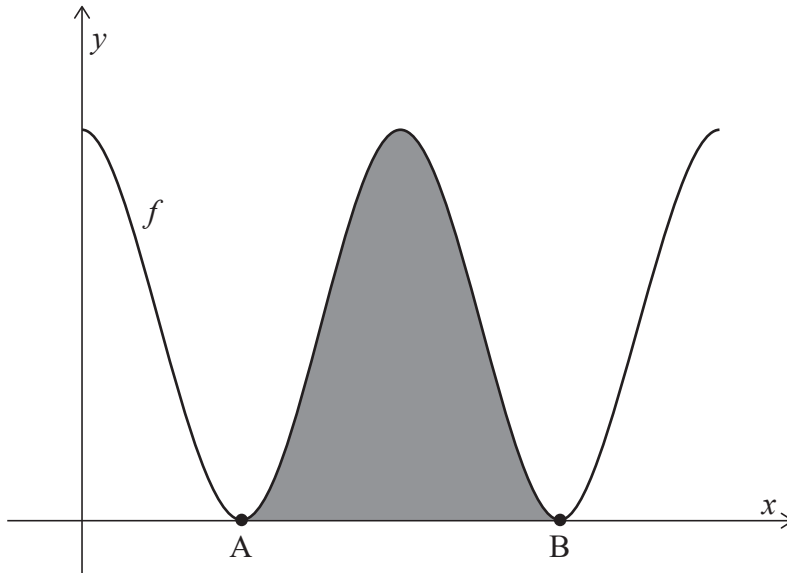


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8. [Maximum mark: 15]

Consider the function f defined by $f(x) = 6 + 6 \cos x$, for $0 \leq x \leq 4\pi$.

The following diagram shows the graph of $y = f(x)$.



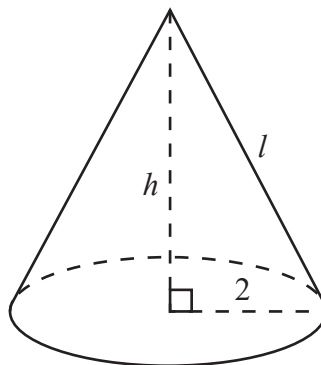
The graph of f touches the x -axis at points A and B, as shown. The shaded region is enclosed by the graph of $y = f(x)$ and the x -axis, between the points A and B.

- (a) Find the x -coordinates of A and B. [3]
- (b) Show that the area of the shaded region is 12π . [5]

The right cone in the following diagram has a total surface area of 12π , equal to the shaded area in the previous diagram.

The cone has a base radius of 2, height h , and slant height l .

diagram not to scale



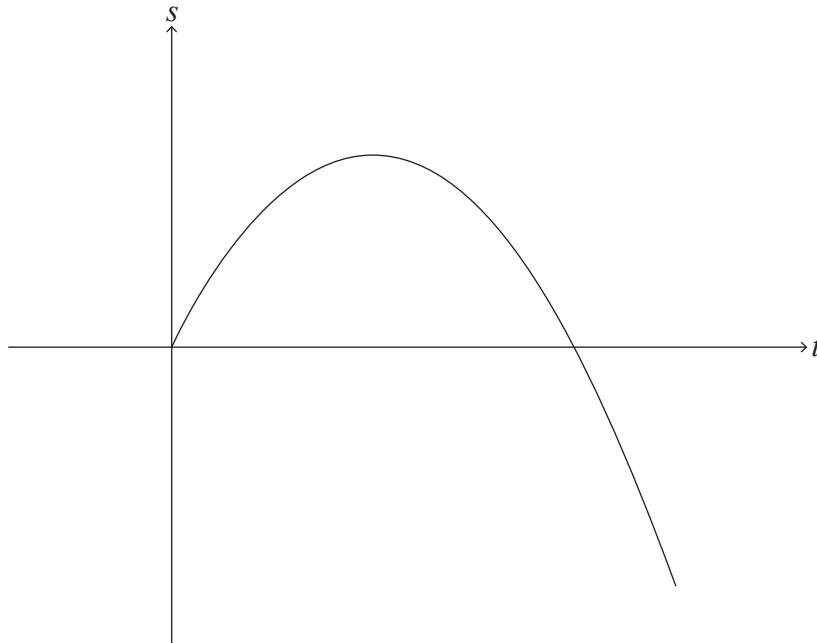
- (c) Find the value of l . [3]
- (d) Hence, find the volume of the cone. [4]



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9. [Maximum mark: 14]

Particle A travels in a straight line such that its displacement, s metres, from a fixed origin after t seconds is given by $s(t) = 8t - t^2$, for $0 \leq t \leq 10$, as shown in the following diagram.



Particle A starts at the origin and passes through the origin again when $t = p$.

- (a) Find the value of p . [2]

Particle A changes direction when $t = q$.

- (b) (i) Find the value of q .
- (ii) Find the displacement of particle A from the origin when $t = q$. [4]

- (c) Find the distance of particle A from the origin when $t = 10$. [2]

The total distance travelled by particle A is given by d .

- (d) Find the value of d . [2]

A second particle, particle B, travels along the same straight line such that its velocity is given by $v(t) = 14 - 2t$, for $t \geq 0$.

When $t = k$, the distance travelled by particle B is equal to d .

- (e) Find the value of k . [4]

References:



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Answers written on this page
will not be marked.



12EP12